Optimization of the Ship Hull Hydrodynamic Characteristics in Calm Water

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The paper discusses the application of a genetic algorithm, coupled with a three-dimensional potential flow solver, in order to deal with the hydrodynamic optimization of a ship hull form with a bulbous bow. The bulb is defined by several geometrical parameters and a set of splines in tension has been used to describe the ship hull form and to allow the modifications of the ship forebody. The optimization procedure is fully automated, and once the basis hull form, objective function, optimization parameters and constrains are defined, it requires no user interaction. According to the predefined constraints, optimized hull forms have been obtained, and hence analyzed and compared to the basis hull form in order to demonstrate the effectiveness and validity of the developed procedure.

Keywords: *genetic algorithm, hydrodynamic optimization, ship hull, spline in tension*

Optimizacija hidrodinamičkih značajki forme broda u mirnoj vodi

Izvorni znanstveni rad

U radu je prikazana primjena genetskog algoritma, koji je povezan s programom za trodimenzijsko rješenje potencijalnog strujanja, u svrhu hidrodinamske optimizacije forme broda s pramčanim bulbom. Pramčani bulb je definiran s više geometrijskih parametara, a skupina napetih *splajnova* korištena je za opisivanje forme broda te za omogućavanje izmjena u pramčanom dijelu. Optimizacijski postupak je potpuno automatiziran te nakon definiranja početne forme broda, funkcije cilja, optimizacijskih parametara i ograničenja, daljnja interakcija korisnika nije potrebna. U skladu s prethodno odabranim ograničenjima, dobivene su optimizirane forme broda koje su zatim analizirane i uspoređene s početnom kako bi se pokazala učinkovitost i valjanost razvijenog postupka.

Ključne riječi: *forma broda, genetski algoritam, hidrodinamska optimizacija, napeti splajn*

1 Introduction

The knowledge of the flow around the ship hull and its hydrodynamic characteristics are very important in ship hull form design. The ship design is a complex process where compromises must be made among various and often contradictory requirements. The problem can be formulated as determination of a set of design variables subject to certain relations between variables and restrictions of these variables. In general, many factors should be considered and not all of them are of hydrodynamic nature. But, from the hydrodynamic point of view, the most interesting goal is to obtain a ship hull with enhanced hydrodynamic characteristics. In order to perform hydrodynamic optimization, an objective function that compares the merit of different designs quantitatively needs to be defined. This objective function depends on design variables and the changes in flow variables due to them. The aim is then to minimize this objective function subject to equations that govern the flow and geometry constraints. The CFD-based hull-form hydrodynamic optimization consists in a CFD solver that can be used to compute the flow field and evaluate the objective function required by the optimization technique, hull geometry modeling and modification that are linked to the design variables and the optimization techniques that can be used to minimize the objective function under given constraints. Many interesting works on hull form optimization have been presented through the years [1, 2, 3, 4, 5, 6, 7].

In the paper, a hydrodynamic optimization problem for a ship hull has been treated and in order to deal with this problem, a genetic algorithm technique has been used. A non-interactive procedure has been developed to minimize an objective function, related with the flow past a ship moving with steady forward speed in calm water. The flow solver coupled with the genetic algorithm is based on a free surface potential formulation. The developed procedure is used for the hydrodynamic optimization of the fore part of the ship hull and the computational examples are presented in order to demonstrate the effectiveness and validity of the developed procedure.

2 The optimization problem formulation

Many optimization problems may be generally formulated as problems of minimizing an objective function $f(\vec{\xi})$ of a number of variables ξ_1, ξ_2, ξ_3,..., ξ_n subject to a group of constraints that can be formulated as equalities or inequalities.

The solution of the optimization problem calls for the formulation of a suitable optimization procedure. Therefore, a potential flow solver [8] and a genetic algorithm [9] have been coupled to build a procedure for the bulbous bow optimization. The optimization procedure starts from a specified basis hull form, by calculating the flow past the hull form and evaluating the wave resistance. The genetic algorithm creates an initial population of specified number of individuals (hull shapes), randomly generated within upper and lower bounds for the optimization parameters.

The bow geometry modification algorithm is an integral component of the optimization procedure. It allows to obtain the changes of the bulb shape and to automatically remesh the fore part of the hull form for each design case. In every generation, each individual is evaluated using a fitness function and assigned a fitness value. The fitness of an individual is determined calling the potential flow solver. Based on their relative fitness values, individuals in the current population are selected for reproduction. Based upon genetic and evolutionary principles, the genetic algorithm repeatedly modifies the population of artificial individuals.

Generating a new generation, individuals in its current population are improved by performing genetic algorithm operators. The process continues until the specified number of generations is attained and acceptable or the best possible solution evolves. The developed procedure is fully automatic and no user interference is needed during the optimization.

There are several important aspects which need to be set when using a genetic algorithm. The objective function needs to be defined, and the genetic representation must be defined and implemented, as well as the genetic operators. For the presented optimization problem the real coding has been chosen.

The results presented in the study have been carried out by means of a genetic algorithm employing the raw fitness linear scaling, the stochastic uniform sampling as selection operator, the two-point crossover as crossover operator and the multi-bit mutation as mutation operator. In addition, the following genetic algorithm parameters have been adopted:

- String length = 7
- Crossover probability $p_c = 0.5$
- Mutation probability $p_m = 0.3$
- Population size = 40
- Number of generations = 50.

As geometrical constraints, the design waterline was kept the same as in the basis form, while the stem profile and the shape of the fore part were allowed to change. To do so, a set of design variables in terms of x, y and z coordinates, was taken as the required set of variables ξ_1, ξ_2, ξ_3,..., ξ_n introduced previously. The number of design variables n treated in the optimization procedure must remain within some reasonable range. On the other hand, the grid used in computation must capture the ship geometry appropriately in order to resolve changes in the flow with sufficient resolution.

In order to obtain a wide range of changes with the lowest possible number of parameters, a set of splines in tension has been applied to describe the geometry of the bulbous bow.

3 The numerical model

The flow around a ship hull advancing in calm water with a constant velocity U is considered. The origin of the coordinate system is located at the fore perpendicular at the level of the undisturbed water plane =0. The x-axis is pointing astern, the y-axis to starboard and z-axis vertically upward. The steady potential flow assumption is made, and a velocity potential ϕ is introduced and the field equation to be satisfied is the Laplace equation:

$$\mathrm{div}(\mathrm{grad}\phi) = \nabla(\nabla\phi) = \Delta\phi \,. \tag{1}$$

On the wetted surface of the hull, the following kinematic boundary condition is imposed:

$$\vec{n}\cdot\nabla\phi = 0, \tag{2}$$

where $\vec{n}$ represents the normal vector on the wetted surface.

On the free surface $z=h$ (x,y), the velocity potential needs to satisfy the kinematic and dynamic boundary condition:

$$\nabla\phi\cdot\nabla h - \partial_z\phi = 0 \;, \tag{3}$$

$$\tfrac{1}{2}(\nabla\phi)^2 + gh - \tfrac{1}{2}U^2 = 0 \;, \tag{4}$$

where g is gravitational acceleration. It is also required that the flow is undisturbed far away from the ship and that the waves created by the ship do not propagate ahead (radiation condition).

The problem is solved by an iterative procedure, and the nonlinear solution is obtained by solving a series of linearized problems, such that the solution of the $(k$-$1)^{\text{th}}$ iteration represents the starting point of the k^{th} iteration. Therefore, the velocity potential ϕ_k of the k^{th} iteration can be expressed as the sum of the potential from the previous iteration ϕ_{k-1} and the perturbation potential $\delta\phi$:

$$\phi_k = \phi_{k-1} + \delta\phi . \tag{5}$$

Accordingly, the wave elevations h_k shown in Figure 1 can be defined:

$$h_k = h_{k-1} + \delta h.. \tag{6}$$

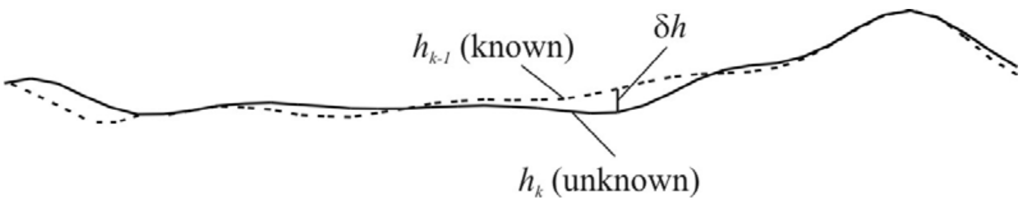

Figure 1 **Free surface elevations**
Slika 1 **Elevacije slobodne površine**

The velocity is defined as the sum of the inflow velocity and the perturbation velocity due to the flow past the hull form:

$$\nabla\phi_k = U\vec{i} + \nabla\phi' . \tag{7}$$

When all the considerations from equations (1) to (7) are taken, the boundary condition on the free surface can finally be written as:

$$\begin{aligned} &\nabla\phi_{k-1}\cdot\nabla(\nabla\phi_{k-1}\cdot\nabla\phi') - g\nabla\phi'\cdot\nabla h_{k-1} + g\partial_z\delta\phi = \\ &= \nabla\phi_{k-1}\cdot\nabla(\nabla\phi_{k-1}\cdot\nabla\phi_{k-1} - U_\infty\partial_x\phi_{k-1}\nabla\phi') + gU_\infty\partial_x h_{k-1} \end{aligned}, \tag{8}$$

where

$$\begin{aligned} &\nabla\phi_{k-1} = \partial_x\phi_{k-1}\,\vec{i} + \partial_y\phi_{k-1}\,\vec{j} + \partial_z\phi_{k-1}\,\vec{k}, \\ &\nabla\phi' = \partial_x\phi'\,\vec{i} + \partial_y\phi'\,\vec{j} + \partial_z\phi'\,\vec{k}, \\ &\nabla h_{k-1} = \partial_x h_{k-1}\,\vec{i} + \partial_y h_{k-1}\,\vec{j}. \end{aligned} \tag{9}$$

The shape of the wetted and of the free surface is adjusted from one iteration to the other. As convergence criterion, the difference between all wave elevations from the last two iterations is set to be less than $|0.0001\ L_{PP}|$, where L_{PP} is ship length between perpendiculars.

The above formulation is solved numerically, and a panel method is implemented through the in-house made software in Fortran code.

The wetted surface and the free surface are discretized into panels, as shown in Figure 2. In order to obtain the automatic remeshing of the wetted surface in each iteration (Figure 2

a) and b)), and during the optimization procedure as well, the hull form is described with a series of splines in tension, paying special attention to the description of the forebody.

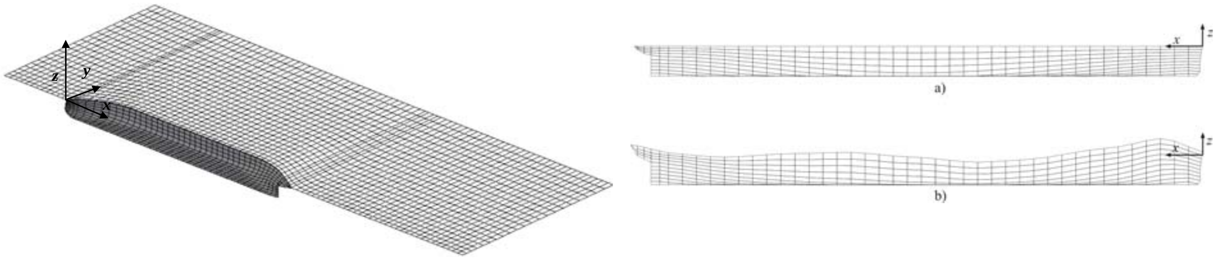

Figure 2 **Discretization of the wetted and free surface**
Slika 2 **Diskretizacija oplakivane i slobodne površine**

4 The spline in tension

In order to allow the modifications of the bulbous bow defined by several parameters [10], the spline in tension has been applied to describe the geometry of the bulb. The splines in tension were first introduced by Schweikert [11]. These curves behave smoothly through the data points with a minimum number of oscillations or inflection points. By varying the tension factor σ, the constructed curve can take different shapes:

a) If $\sigma \to 0$, the curve converges to a cubic spline
b) If $\sigma \to \infty$, the curve converges to a piecewise linear curve, with still sharp but rounded corners.

The main advantages of the use of tension splines for the representation of the bulbous bow are the fairness and the robustness of the curves. Also, the shape of the curve can be altered by simply varying the tangent angles in the first or last point of the curve, Figure 3.

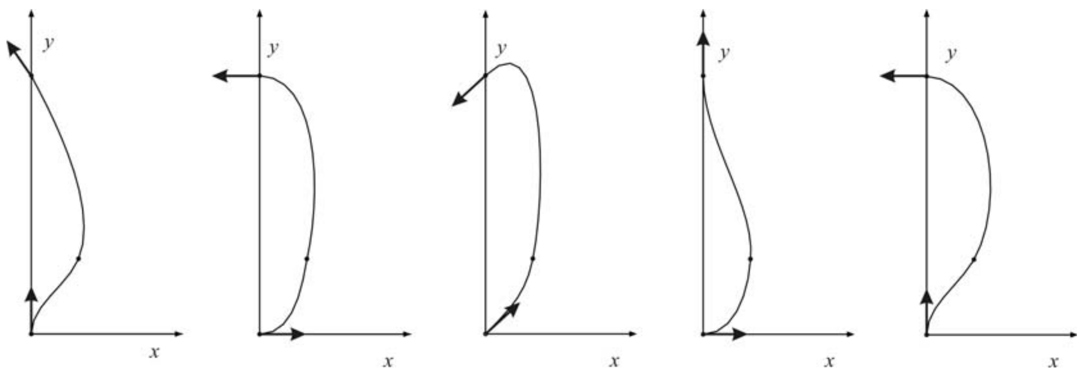

Figure 3 **The influence of the tangent angle on the shape of the spline in tension**
Slika 3 **Utjecaj kuta tangente na oblik napetog splajna**

To create a spline in tension, a set of points $x_1, x_2, \ldots, x_n$ must be given, with the corresponding values of $y_1, y_2, \ldots, y_n$. This formulation requires the values on the abscissa to be strictly increasing ($x_i < x_{i+1}$), otherwise a parametric formulation must be used. A real-valued function f needs to be found, with a continuous first and second derivative, and such that:

$$f(x_i) = y_i \text{, for each } i = 1, \ldots n. \tag{10}$$

It is also required the quantity $\left(f''-\sigma^2 f\right)$ to vary linearly on each of the intervals $\left[x_i, x_{i+1}\right]$, i = 1,…, n-1. For each x, such as $x_i \le x \le x_{i+1}$, the following equation can be set:

$$\begin{aligned} f''(x)-\sigma^2 f(x) &= \left[f''(x_i)-\sigma^2 y_i\right]\cdot\left(x_{i+1}-x\right)/h_i + \\ &+\left[f''(x_{i+1})-\sigma^2 y_{i+1}\right]\cdot\left(x-x_i\right)/h_i \end{aligned} . \tag{12}$$

where $h_i = x_{i+1}-x_i$, for i = 1,…, n-1.

Solving (2), and invoking conditions (1), result in:

$$\begin{aligned} f(x) &= \left[f''(x_i)/\sigma^2\right]\cdot\sinh\left[\sigma\left(x_{i+1}-x\right)\right]/\sinh\left(\sigma h_i\right)+ \\ &+\left[y_i - f''(x_i)/\sigma^2\right]\cdot\left(x_{i+1}-x\right)/h_i + \\ &+\left[f''(x_{i+1})/\sigma^2\right]\cdot\sinh\left[\sigma\left(x-x_i\right)\right]/\sinh\left(\sigma h_i\right)+ \\ &+\left[y_{i+1}-f''(x_{i+1})/\sigma^2\right]\cdot\left(x-x_i\right)/h_i \end{aligned} \tag{13}$$

The above formulation of the spline in tension is not applicable for any set of points [12], for example closed curves or curves with the tangent parallel to the y-axis at some point. Such problem can be overcome by introducing the parametric form of the spline in tension [13], which requests to adopt the curvilinear abscissa as independent variable. The additional variable s is defined as:

$$\begin{aligned} &s_i = 0,\ i=1 \\ &s_i = s_{i-1}+\sqrt{\left(x_i-x_{i-1}\right)^2+\left(y_i-y_{i-1}\right)^2},\ i=2,n \end{aligned} , \tag{14}$$

where s is the arc length of the polygonal passing through the given points.

If the parametric form of the spline in tension is applied, the linear system of equations is similar to the previously introduced, except for the variable which in this case is s instead of x or y.

To describe the fore part of the hull form, three splines in tension have been used, as shown in Figure 5: one to describe the waterline, which changes it shape according to the wave profile along the hull (spline S1), one for the stem profile (spline S2) and one for the maximum beam line of the bulb (spline S3). The maximum beam line refers to a line connecting the points in which beam modifications are allowed. The design variables, ξ_1, ξ_2, ξ_3,..., ξ_{12}, allow modifications in the x and z direction on the stem profile, and modifications in the y and z direction on the maximum beam line.

The allowed range of variation for the considered variables is given in Table 1. The range is quite large on purpose, to see towards what shape the bulb would be developed. In this case no limitations were imposed regarding the ship displacement volume, wetted area or position of the center of buoyancy. But from the ship designer's point of view, such limitations should be strictly taken care of. The developed procedure allows setting the variables according to the problem at hand, so any ship's hull characteristic can be introduced into the optimization procedure, as fixed or variable parameters.

Table 1 **Variables range**
Tablica 1 **Raspon varijabla**

Variable	Range of variation, m
ξ_1	$-3.0 \le x \le 0.0$
ξ_2	$-1.0 \le z \le 1.0$
ξ_3	$-0.5 \le x \le 1.5$
ξ_4	$-0.5 \le y \le 1.5$
ξ_5	$-0.5 \le x \le 1.5$
ξ_6	$-0.5 \le y \le 2.0$
ξ_7	$-1.0 \le x \le 2.0$
ξ_8	$-1.0 \le y \le 2.0$
ξ_9	$-1.0 \le z \le 2.0$
ξ_{10}	$-1.0 \le x \le 0.5$
ξ_{11}	$-1.0 \le z \le 0.5$

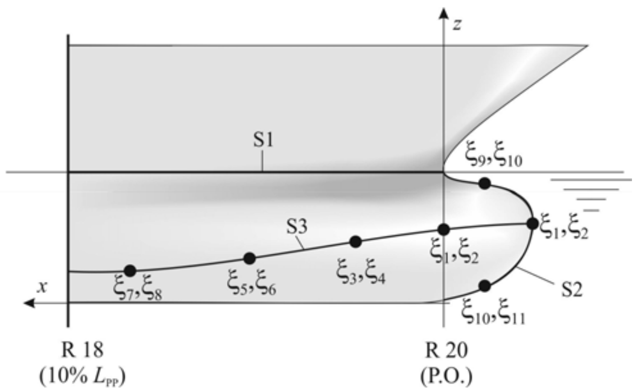

Figure 4 **The variables**
Slika 4 **Varijable**

5 Results

The optimization procedure has been applied to a tanker hull form, taken as the basis hull form. The principal ship particulars are the following: length between perpendiculars = 169.0 m, breadth = 32.0 m, draught = 12.0 m, displacement = 43506 t, block coefficient = 0.785, midship coefficient= 0.985.

The fore part of the hull form has been optimized for a single speed corresponding to the Froude number 0.18, based on L_{PP}. Two separate cases of the optimization procedure have been treated. In the first case, the wave resistance was set as object function, while in the second case the considered object function was the root sum square of the wave elevations.

5.1 The wave resistance as object function

In the first case, the optimization was run with setting the wave resistance as object function. The wave resistance R_W is calculated as:

$$R_W = 0.5 \rho C_W U^2 S , \quad (15)$$

using the wave resistance coefficient C_W obtained by integrating the x-components of the pressure forces acting on the submerged portion of the hull form. In (15) ρ represents the water density in kg/m^3, U *is* the ship speed in m/s and S is the wetted surface in m^2.

The evolution history of wave resistance for the evaluated hull forms is presented in Figure 5. In the figure each dot represents the absolute value of the wave resistance for each individual (hull shape) evolved during the optimization. Having 40 individuals through 50 generations, a total of 2000 evaluation cycles have been executed before the optimal hull form emerged. The solid line represents the optimal solutions front over the generations. It is formed by minimal values of the wave resistance obtained in the simulations. As demonstrated, generation by generation a set of solutions has converged as the given number of generations is gained.

The optimized bulb shape is compared to the initial bow shape in Figure 6. The resulting bow shape is entirely dictated by the hydrodynamic behavior associated with the changes in the bow sections shape. The fore sections are considerably wider than the original, but this is due to the choice of the optimization parameter range.

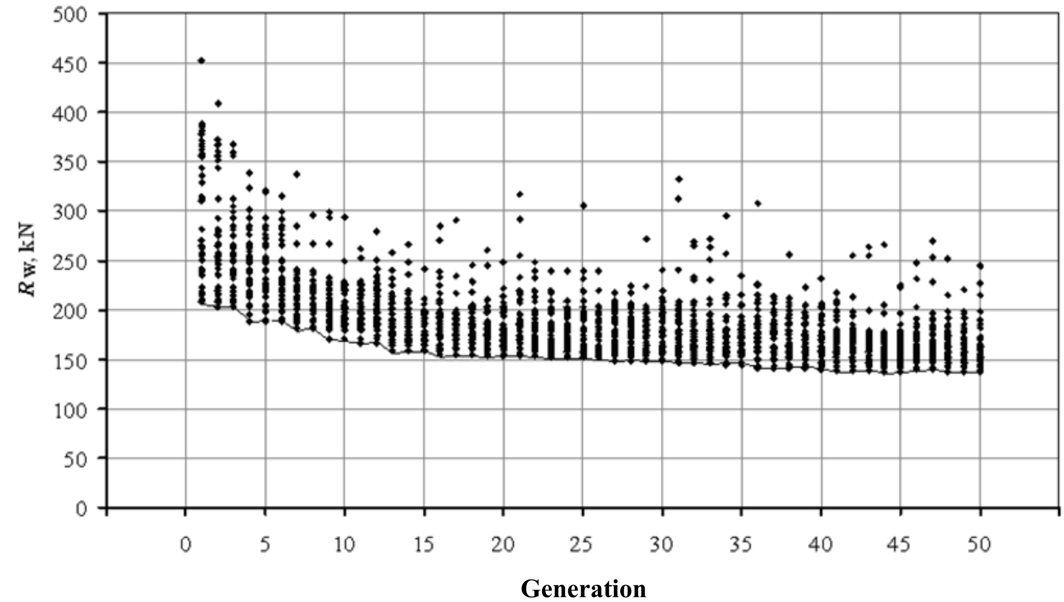

Figure 5 **Evolution history of the optimum ship hull form for the R_W set as object function**
Slika 5 **Evolucija optimalne brodske forme, R_W je postavljen kao funkcija cilja**

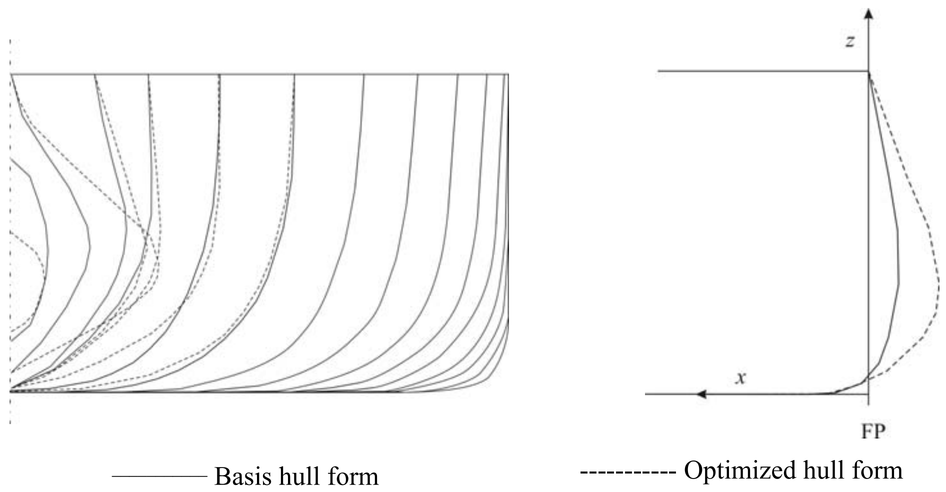

Figure 6 **Comparison of the basis and optimized hull form**
Slika 6 **Usporedba početne i optimizirane forme**

Figure 7 presents the comparison for the wave profile for the basis hull form and optimized hull form advancing at Fr=0.18.

Both the hull length and the wave elevations are normalized by $L_{PP}/2$. Wave elevations h are plotted for the collocation points of the panels next to the centerline (y/L_{PP}=0.0), when $2x/L_{PP}$ <-1.0 or $2x/L_{PP}$>1.0, and for the panels next to the hull when $-1<2x/L_{PP}<1.0$. A certain wave elevation reduction can be noticed near the fore perpendicular.

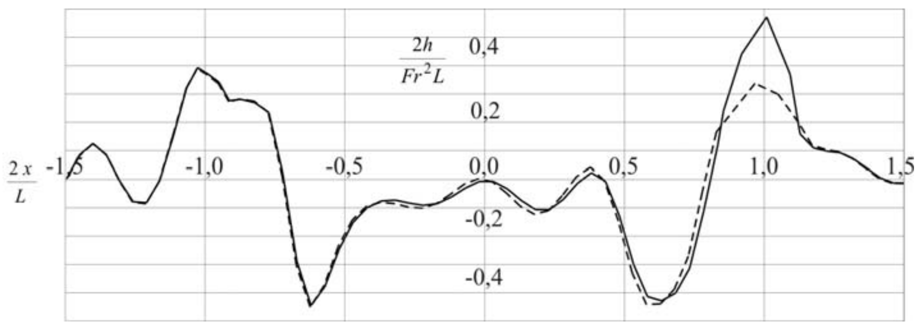

Figure 7 **Wave profile at *Fr* = 0.180**
Slika 7 **Profil vala za *Fr* = 0.180**

Finally, although the problem is treated as a single point design problem, additional numerical tests have been performed with the obtained hull form for Froude numbers ranging from 0.15 to 0.21. The predicted wave resistance values are summarized in Table 2 and Figure 8. Comparing the optimized hull form to the basis one, it is evident that the reduction of the wave resistance coefficient has been achieved over a wide range of Froude numbers. For the design speed, corresponding to *Fr* =18, a wave resistance reduction of 12% can be noticed, while for higher speeds the wave resistance reduction is even higher.

Table 2 Comparison of the wave resistance values
Table 2 Usporedba vrijednosti otpora valova

	Basis hull form	Optimized hull form
Fr	R_W, kN	R_W, kN
0.150	63.800	58.752
0.160	66.278	59.890
0.170	79.294	69.507
0.180	120.119	96.621
0.190	200.950	149.232
0.200	291.820	201.231
0.210	365.864	257.804

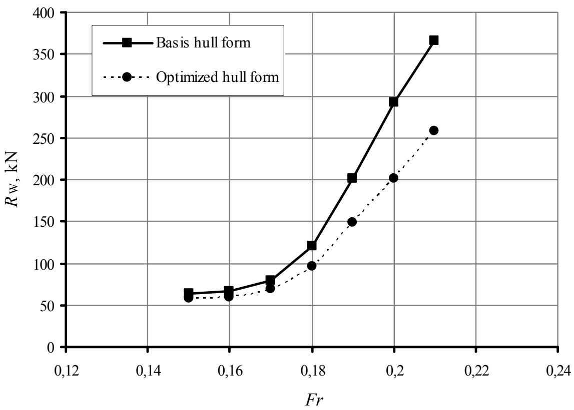

Figure 8 **Comparison of wave resistance versus Froude number**
Slika 8 **Usporedba otpora valova u funkcji Froudeovog broja**

5.2 The wave elevations as objective function

For the other approach, the root sum square (RSS) of the wave elevations in the bow area has been selected as objective function, and defined as follows:

$$f(\vec{\xi}) = \sqrt{\sum_i h_i^2} \quad , \tag{16}$$

where h_i is the wave elevation for the *i*-th free surface panel located in the circular area of the free surface with the centre on the fore perpendicular and the radius equal to 30% L_{PP}. This approach is based upon the influence of the wave elevations on the hydrodynamic characteristics of the hull. The evolution of the optimized full form is shown in Figure 9, where the convergence of the process can be observed.

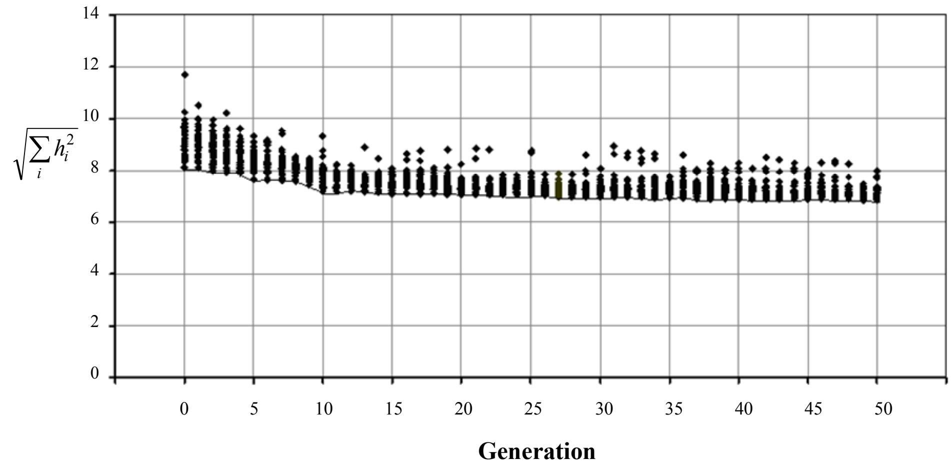

Figure 9 **Evolution history of the optimum ship hull form for the wave elevations RSS set as object function**
Slika 9 **Evolucija optimalne brodske forme, valne elevacije su postavljene kao funkcija cilja**

The sections of the basis and the optimized hull form are presented in Figure 10, together with the obtained stem profile. In Figure 11 the wave profile is shown, and it refers to the panels along the hull. In the bow area a reduction of the wave elevations is visible, if compared to the basis hull form. As in the previous case, the hull form is altered only in the bow area, so there is no change in the appearance of the wave profile in the aft.

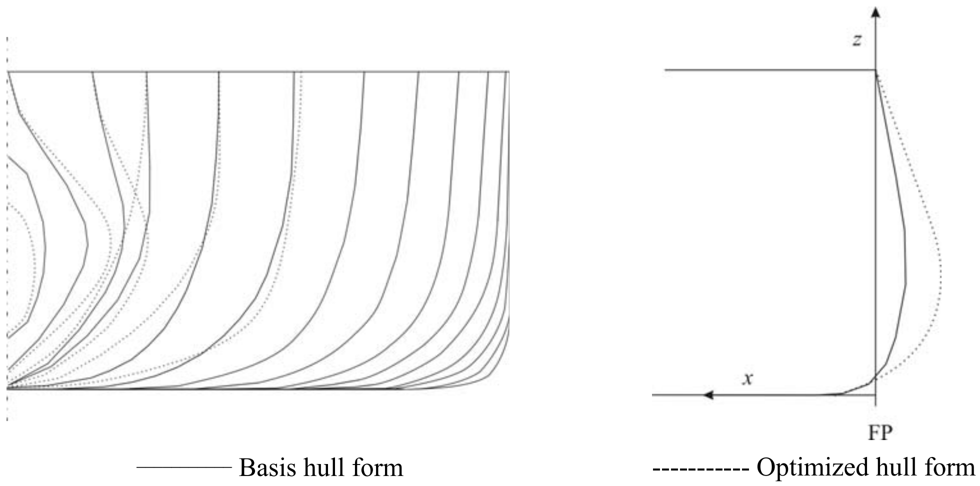

Figure 10 **Comparison of the basis and optimized hull form**
Slika 10 **Usporedba početne i optimizirane forme**

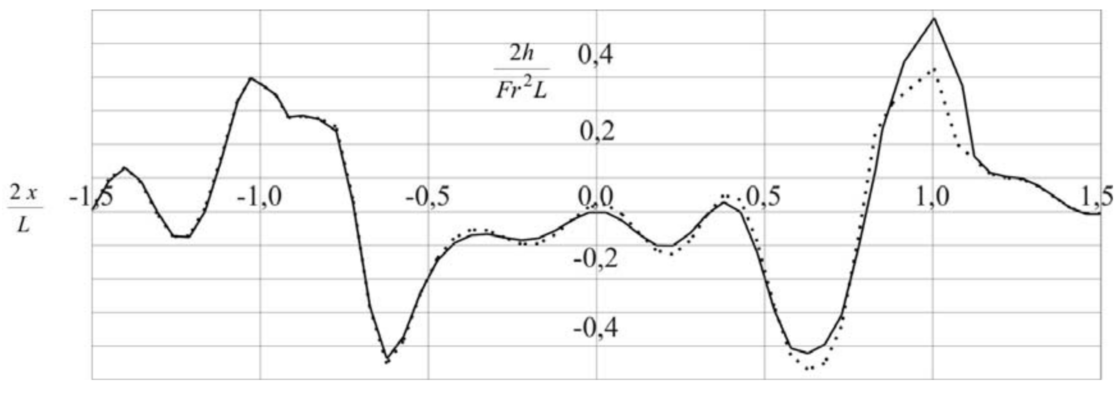

Figure 11 **Wave profile at *Fr* = 0.180**
Slika 11 **Profil vala za *Fr* = 0.180**

In Table 3, the values obtained in the optimization process are shown. Figure 12 shows the root sum square (RSS) of the wave elevations. Reductions are obtained only at Froude numbers higher than 0.17, but since the treated speed corresponds to Fr=0.18, the objective function has been reached. In Figure 13 the wave resistance values are presented. The curve clearly shows the wave resistance for the entire considered speed range. It can be seen that the values of the wave resistance are reduced only at higher speeds, while at lower speeds they are increased. Also, the reduction of the wave resistance is lower than in the previous case, where the wave resistance was selected as the objective function. It is evident that the results of the hydrodynamic optimization process depend greatly on the chosen objective function, as well as on the predefined range of optimization parameters.

Table 3 **Comparison of the results**
Tablica 3 **Usporedba rezultata**

	Basis hull form		Optimized hull form	
Fr	$\sqrt{\sum_i h_i^2}$, m	R_W, kN	$\sqrt{\sum_i h_i^2}$, m	R_W, kN
0.150	3.854	63.800	3.874	73.552
0.160	4.418	66.278	4.450	83.014
0.170	5.118	79.294	5.093	100.164
0.180	5.955	120.119	5.866	122.292
0.190	6.749	200.950	6.620	180.108
0.200	6.968	291.820	6.827	264.971
0.210	8.157	365.864	7.962	322.386

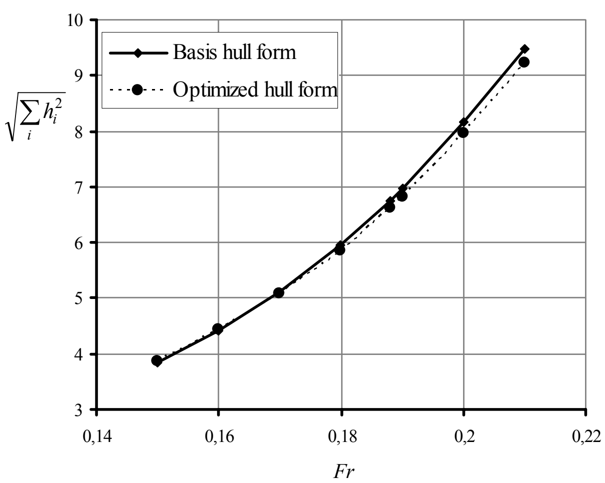

Figure 12 **Comparison of the RSS of wave elevations**
Slika 12 **Usporedba korijena sume kvadrata valnih elevacija**

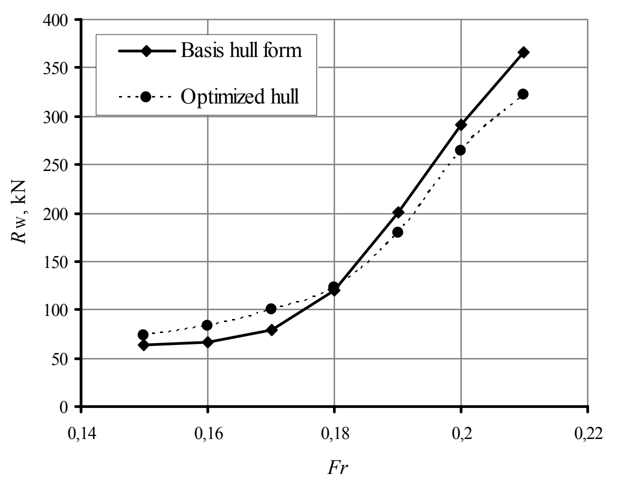

Figure 13 **Comparison of the wave resistance values**
Slika 13 **Usporedba vrijednosti otpora valova**

6 Conclusion

A numerical procedure for ship hull from optimization from the hydrodynamic point of view has been established. The procedure is based on the genetic algorithm and a potential flow solver. The application of the procedure has been illustrated in the case of the tanker hull form. Two single objective functions have been considered: the wave resistance of the ship and the wave elevations in the bow area.

The study has shown that the developed optimization procedure can be successfully applied to the optimization of the fore part of the ship hull and used as a valuable method to favorably modify a ship hull. The procedure requires clearly set goals, i.e. both the objective functions and the constraints should be carefully chosen in order to obtain the hull form with the enhanced hydrodynamic characteristics.

Further work is intended to be done for the application of the developed optimization procedure, and a multi-objective optimization would be considered together with different design variables and geometrical constraints.

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